Local Discriminant Wavelet Packet Basis for Texture Classification

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Abstract

Wavelet packets are well-known for their ability to compactly represent textures consiting of oscillatory patterns such as fingerprints or striped cloth. In this paper, we report recent work on representing both periodic and granular types of texture using adaptive wavelet basis functions. The discrimination power of a wavelet packet subband can be defined as its ability to differentiate between any two texture classes in the transform domain, consequently leading to better classification results. The problem of adaptive wavelet basis selection for texture analysis can, therefore, be solved by using a dynamic programming approach to find the best basis from a library of orthonormal basis functions with respect to a discriminant measure. We present a basis selection algorithm which extends the concept of 'Local Discrminant Basis' (Saito and Coifman, 1994) to two dimensions. The problem of feature selection is addressed by sorting the features according to their relevance as described by the discriminant measure, which has a significant advantage over other feature selection methods that both basis selection and reduction of dimensionality of the feature space can be done simultaneously. We show that wavelet packets are good at representing not only oscillatory patterns but also granular textures. Comparative results are presented for four different distance metrics: Kullback-Leibler (KL) divergence, Jensen-Shannon (JS) divergence, Euclidean distance, and Hellinger distance. Initial experimental results show that Hellinger and Euclidean distance metrics may perform better as compared to other cost functions.

1 Introduction

Texture classification can be defined as a mapping from a set of input image pixels to a set of class labels. Finding appropriate features from an image containing texture regions is key to utilising textural properties which can differentiate between different textures well. Recently, subband filtering methods have been shown [10] to be quite effective in characterising different types of textures. Out of these methods, some have employed wavelet bases, fixed dyadic wavelet basis [7] as well as adaptive wavelet packet basis [1, 3], to represent the image in such a way that discriminant features of underlying textures are highlighted. A major limitation of wavelet representation, however, is that only a restricted subset of possible space-frequency tilings is used to extract spatial frequency components present in the image. Wavelet packets provide a solution to this problem so that adaptive frequency segmentation can be found for a given image based on a specific criterion or a cost function.

In the context of texture classification, the cost function used to find the best wavelet packet basis should be able to provide an estimate of the discrimination power of a subband. Only then can it be ensured that the resulting basis will be an optimal choice from a library of available bases. Chang and Kuo [3] suggested using l_1-norm as a cost function for tree prunning in a top-down manner. A subband is further decomposed only if its l_1-norm is larger than a factor of the maximum norm value at the same resolution. This approach leads to an adaptive tree-structured wavelet decomposition, a term the authors of [3] used for wavelet packet decomposition. Acharyya and Kundu [1] employ an energy based cost function and a top-down search without any decimation to compute the basis wavelet packet basis for texture segmentation. It is to be noted, however, that a top-down search method employed in a best basis selection algorithm cannot guarantee an optimal solution. Laine and Fan [6] used energies of subbands from the full wavelet packet tree as a signature for images belonging to certain texture class.

The *local discriminant basis* (LDB) of Saito & Coifman [12] proposed to use a cost function which can maximise the differences in time-frequency energy distributions of each class. Symmetric Kullback-Leibler (KL) distance was used to measure the dissimilarity between energy distributions of a particular subband for all classes. Another related work is the *local clustering basis* (LCB) of Meyer & Chinrungrueng [8], wherein the authors select basis functions according to their ability to separate the fMRI time-series into activated and non-activated clusters. Locally clustering basis functions are chosen due to their discriminating power, and projections of the raw fMRI data onto these basis functions are computed for efficient segmentation of the data into activated and non-activated regions.

In this paper, we propose a local discriminant wavelet packet basis for texture classification problem as defined above. A locally discriminant set of basis functions orthogonal to eachother is chosen using a bottom-up dynamic programming approach in such a way that the dissimilarities in space-frequency energy distributions are maximised. We investigate four different cost functions used for measuring the discriminating power of a subband and consequently for prunning the full wavelet packet tree to obtain an adaptive LDB for a given image. A subset of most discriminant features is chosen using the discriminating power of a subband (feature) to avoid the so-called curse of dimensionality.

The paper is organised as follows. An overview of wavelets and wavelet packets for texture analysis is provided in the next section. Algorithm for local discriminant basis selection alongwith a description of four different cost functions is provided in Section 3. Experimental settings and results are presented in Section 4 and the paper concludes with remarks on the results and further directions for research.

2 Wavelet Packets

2.1 Introduction

As opposed to Fourier basis functions, the principle behind wavelets is that shifts and dilations of a prototype function $\psi(t)$ are chosen as basis functions, decomposing the signal into its components belonging to different frequencies while providing good localization in time (space) at the same time. The discrete wavelet transform can be computed with the help of filter banks that decompose the signal (image) into low and high frequency subbands. The low frequency subband is further decomposed in order to go down the transform one more level. Wavelet based texture classification methods use the wavelet subbands to extract textural features – see [7, 10] for a review of these methods. Contributions of each subband to the image (ie, frequency components of the image corresponding to that subband) are usually passed through a nonlinearity followed by a smoothing function to compute a feature image.

2.2 Wavelet Packet Decomposition

A more general form of the wavelet basis, known as the *wavelet packet basis* [4] adaptively segments the frequency axis based on a certain cost function. The frequency intervals of varying bandwidths are adaptively selected to extract specific frequency contents present in the given signal. This frequency segmentation is useful, for example, to analyze a local phenomenon occurring in the signal and belonging to a specific frequency band. The discrete wavelet packet transform of a 1-d discrete signal $\mathbf{x} = x_i$, $i = 0, 1, \ldots, N-1$ can be computed as follows. The wavelet packet coefficients are defined as

$$\begin{aligned} w_0^0(l) &= x_l & l &= 0, \ldots, N-1 \\ w_j^{2p}(l) &= \sum_k g_{k-2l}\, w_{j-1}^n(k) & l &= 0, \ldots, N2^{-j}-1 \\ w_j^{2p+1}(l) &= \sum_k h_{k-2l}\, w_{j-1}^n(k) & l &= 0, \ldots, N2^{-j}-1 \end{aligned} \tag{1}$$

where $j = 1, 2, \ldots, J$; $J = \log_2 N$, $w_j^p(l)$ is the transform coefficient corresponding to the wavelet packet function which has relative support size 2^j, frequency $p2^j$ and is located at $l2^j$. In other words, j, p and l can be regarded as the scale, frequency and position indices of the corresponding wavelet packet function respectively. The coefficients $\{h_n\}$ and $\{g_n\}$ correspond to the lowpass and highpass filters respectively for a two-channel filter bank and the transform is invertible if appropriate dual filters $\{\tilde{h}_n\}$, $\{\tilde{g}_n\}$ are used on the synthesis side. When comparing to the wavelet decomposition, it can be regarded as a decomposition which lifts the limit of only decomposing the lowpass filtered signal so that all the highpass subbands can be further decomposed as well. This results in a combinatorial explosion of possible bases which to select a suitable basis from. Since this library of available bases provides an overcomplete representation, a fast optimization algorithm such as [5] is required to select a combination of bases from this library which is well suited to the signal under consideration.

2.3 Wavelet Packet Texture Analysis

In the case of general wavelet packet decomposition, a basis needs to be selected which has the maximum discriminating power among all possible bases in the library of wavelet packets. Although this adds an extra computational cost, a fast dynamic programming algorithm can be used to select an optimal basis. Apart from this, texture classification using wavelet packet subbands may proceed in almost the same way a system based on wavelet subbands works, as described in Section 2.1.

3 Local Discriminant Basis

Coifman & Wickerhauser [5] proposed to use a dynamic programming approach to select the best wavelet packet basis functions that can compactly represent a given signal. To achieve the goal of signal compression, they proposed entropy based cost functions to estimate the information contents of a subband. A subband is preferred on its child subbands if its entropy is less than sum of that of all its child subbands. However, this has little relevance to texture classification as smaller entropy of a subband does not necessarily mean that the subband will prove to be efficient for separating pixels belonging to different classes of textures.

Considering a certain wavelet packet subband for two types of textures as two space-frequency energy distributions, one way of computing the discriminating power of that subband is to find how dissimilar these distributions are. Distance measures for probability distributions can then be used on psuedo-distributions yielding a measure of discrimination power. Let F and G denote the transform coefficients of a particular subband for training images $\mathbf{x}_1$ and $\mathbf{x}_2$, belonging to two different texture classes, respectively. We considered three ways of forming pseudo-distributions f from the subband coefficients F:

$$f_1(x,y) = F(x,y), \qquad f_2(x,y) = |F(x,y)|^2, \qquad f_3(x,y) = |F(x,y)|^2/\|\mathbf{x}_1\|^2 \tag{2}$$

Similarly, the pseudo-distributions g_i $(i = 1, 2, 3)$ can be obtained by using the subband coefficients $G(x,y)$ and texture image $\mathbf{x}_2$. We tested four cost functions in our experiments: symmetric Kullback-Leibler (KL) divergence, Jensen-Shannon (JS) divergence, Euclidean distance (ED), and Hellinger distance (HD) – denoted respectively by KL_i, JS_i, ED_i, and HD_i – defined as follows.

$$KL_i(F,G) = D(f_i||g_i) + D(g_i||f_i), \tag{3}$$

$$JS_i(F,G) = \frac{D(f_i||f_ig_i) + D(g_i||f_ig_i)}{2} \tag{4}$$

$$ED_i(F,G) = \|f_i - g_i\|_2 \tag{5}$$

$$HD_i(F,G) = \sqrt{\sum_x \sum_y \left[\sqrt{f_i(x,y)} - \sqrt{g_i(x,y)}\right]^2} \tag{6}$$

where

$$D(f||g) = \sum_x \sum_y f(x,y) \log \frac{f(x,y)}{g(x,y)}$$

is the relative entropy between f and g,

$$f_ig_i(x,y) = \frac{f_i(x,y) + g_i(x,y)}{2}$$

is the average distribution $\forall x, y$, $\|\cdot\|_2$ denotes the l_2-norm, and $\mu(f)$ and $\sigma^2(f)$ respectively are the mean and variance of f.

Algorithm:

Let $\mathcal{C}(\mathbf{x}_1, \mathbf{x}_2, \mathcal{B})$ denote the cost function, one of the above four, representing the discriminating power of a basis $\mathcal{B}$ in terms of its capability to separate $\mathbf{x}_1$ and $\mathbf{x}_2$. Let $\mathcal{B}_j^{p,q}$ denote the wavelet packet basis for a node $\lambda_j^{p,q}$ of the full wavelet packet tree and let $\mathcal{O}_{j+1}^{2p,2q}$, $\mathcal{O}_{j+1}^{2p,2q+1}$, $\mathcal{O}_{j+1}^{2p+1,2q}$, and $\mathcal{O}_{j+1}^{2p+1,2q+1}$ denote the wavelet packet bases corresponding to four children of the node $\lambda_j^{p,q}$. The best wavelet packet basis up to a depth J, where $J = \log_2 N$ and N is the number of pixels in each dimension, for texture classification is selected as follows.

1. Compute the J-level full wavelet packet tree decomposition.
2. Initialize $j \longleftarrow J-1$.
3. For all $0 \leq p < 2^j$, $0 \leq q < 2^j$, do the following:
   (a) If $\mathcal{C}(\mathbf{x}_1, \mathbf{x}_2, \mathcal{B}_j^{p,q}) > [\mathcal{C}(\mathbf{x}_1, \mathbf{x}_2, \mathcal{O}_{j+1}^{2p,2q}) + \mathcal{C}(\mathbf{x}_1, \mathbf{x}_2, \mathcal{O}_{j+1}^{2p,2q+1}) + \mathcal{C}(\mathbf{x}_1, \mathbf{x}_2, \mathcal{O}_{j+1}^{2p+1,2q}) + \mathcal{C}(\mathbf{x}_1, \mathbf{x}_2, \mathcal{O}_{j+1}^{2p+1,2q+1})]$.

      keep the four child subbands at depth $j+1$,

   otherwise

      merge them to get $\lambda_j^{p,q}$.
3. Decrement j by 1.
4. If $j < 0$, then stop, otherwise go to step 3.

The computational complexity of above algorithm is $O(N \log N)$.

4 Feature Selection

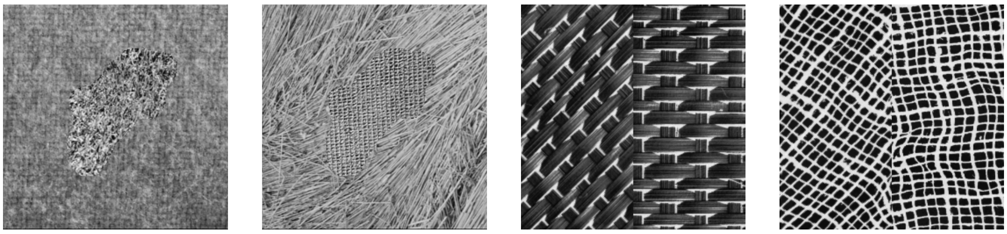
The issue of selection of features from subband decomposition demands more scrutiny now due to a large number of possible bases that can be used to represent the image. Let us define the feature selection problem in the context of subband decomposition as follows. Given a test image $\mathbf{x}$ that has been decomposed into n subbands, each of which can be regarded as a feature, the goal is to select m subbands such that the resulting misclassification error is minimal. We have shown that it is still possible to gauge the discrimination power of a subband independent of the classifier. Given the nature of subband decompositions under consideration, a subband can be regarded as being highly discriminant if it highlights the frequency characteristics of one class but not the other. In other words, if the coefficients of a particular subband light up (ie, are higher in magnitude) for one class but are relatively insignificant for another one, the subband can prove to be helpful in terms of classification performance.

In our previous work [9], we have proposed to use a symmetric KL distance between the normalised energies of a subband of training images as a measure of relevance of an estimate of the discrimination power of the subband. However, Saito et al. [11] warn that the approach of sequentially measuring the efficacy of each dimension of the feature space independently may be "too greedy" as 2D and higher dimensional structures in the feature space may be missed. The principal advantage, however, is that we can make a feature selection solely on training data which reduces the complexity of the final classifier on test samples. When compared with traditional multivariate feature projection methods like PCA or LDA, this advantage is significant.

Once it has been ensured that the basis chosen to represent the image is optimal for texture classification, selection of most discriminant subbands (features) can proceed by using the cost function as a measure of relevance of a feature to classification. Thus cost function values need to be computed only once to be used for basis selection and subsequent feature selection follows by ranking the features according to these values, an approach proposed in [2], saving computations if basis selection and feature selection were done separately.

5 Experimental Results

The basis and feature selection algorithms outlined above were tested on four test images containing textures of granular and periodic nature taken from the Brodatz texture collection: D9D19f (grass/wool), F17D15f (straw/cloth), D65D65R (fence/fence) and D103D103R (burlap/burlap) shown in Figure 1. Classification results for these images using four cost functions and two-level wavelet, full wavelet packet (FWP), and adaptive WP bases are shown in Figures 2–5. Most discriminant features were used with a k-means classifier to assign class labels. From these experiments, it can be seen that our basis selection algorithm tends to find a basis resembling wavelet or FWP geometry whichever of them produces better results. Feature selection enables us to find a relatively small subset of features thus speeding up the computations in many applications. While these results may provide some indication as to which of the four cost functions performs better for a certain type of texture, it is perhaps due to the limited number of experiments that no firm conclusions may be drawn in this regard.



D9D19f	F17D15f	D65D65r	D103D103R
grass/wool	straw/cloth	fence/fence	burlap/burlap
(a)	(b)	(c)	(d)

Figure 1: Test images created from Brodatz collection

6 Conclusions

In this paper, we proposed a fast algorithm for local discriminant basis selection from a library of wavelet packet functions for texture classification. The use of a cost function suitable for measuring the discriminating power of a subband was advocated and four such cost functions were studied. Once the basis is selected, corresponding features can be ranked based on their cost function values. Experiments were restricted to a two-class classification problem. Future research directions include extension of this algorithm to a multi-class problem and incorporation of a sophisticated classifier.

References

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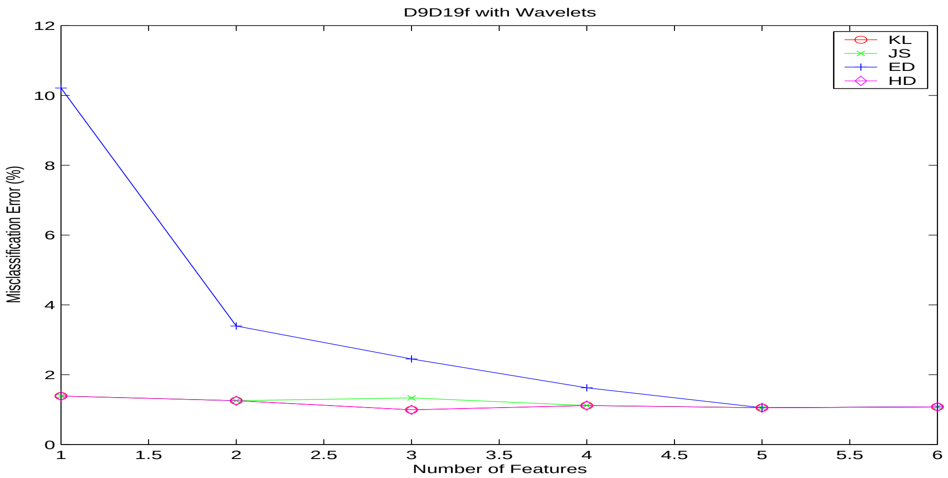

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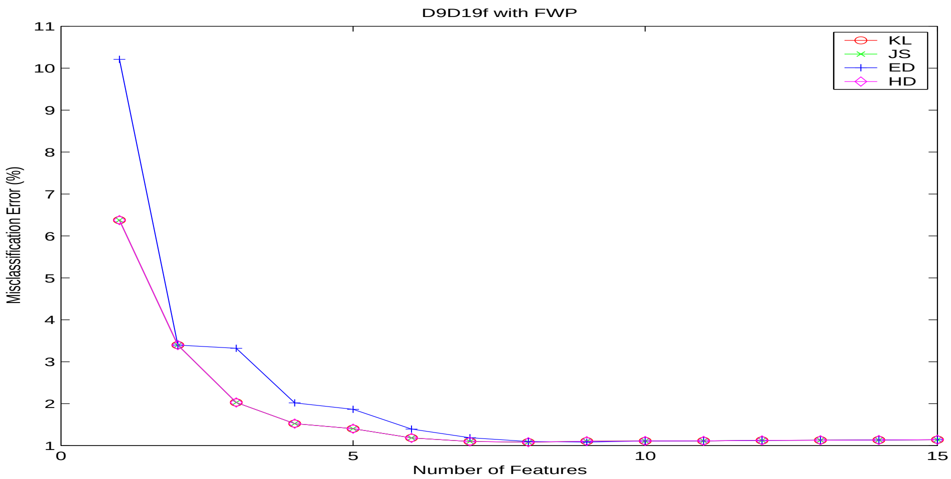

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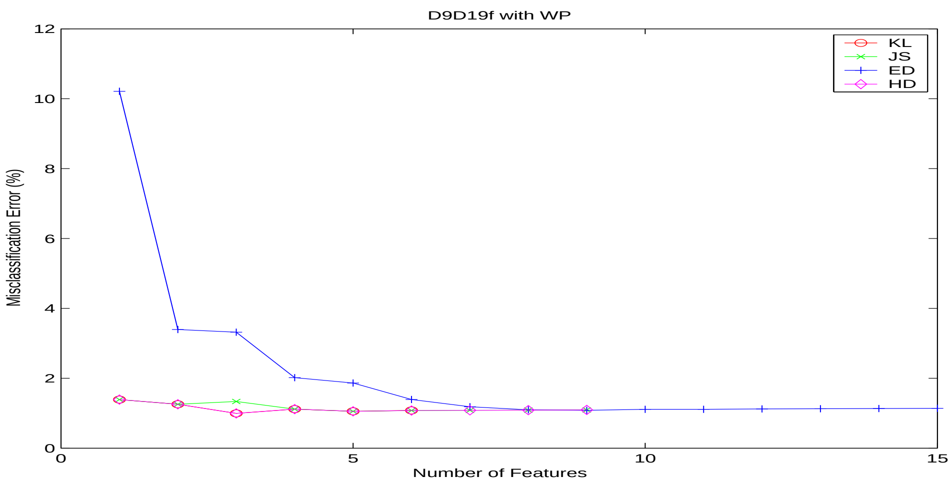
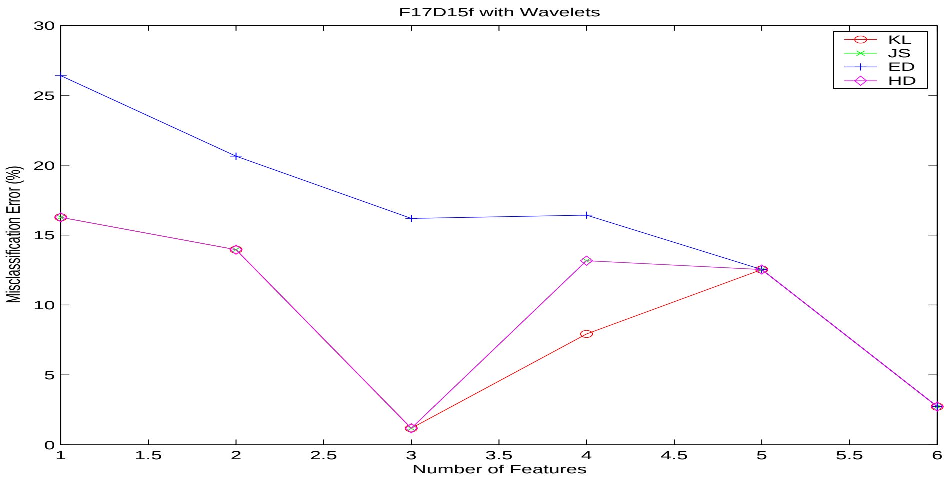


(c)

Figure 2: Classification results for D9D19f
Using (a) Wavelets, (b) FWP, and (c) Adaptive WP



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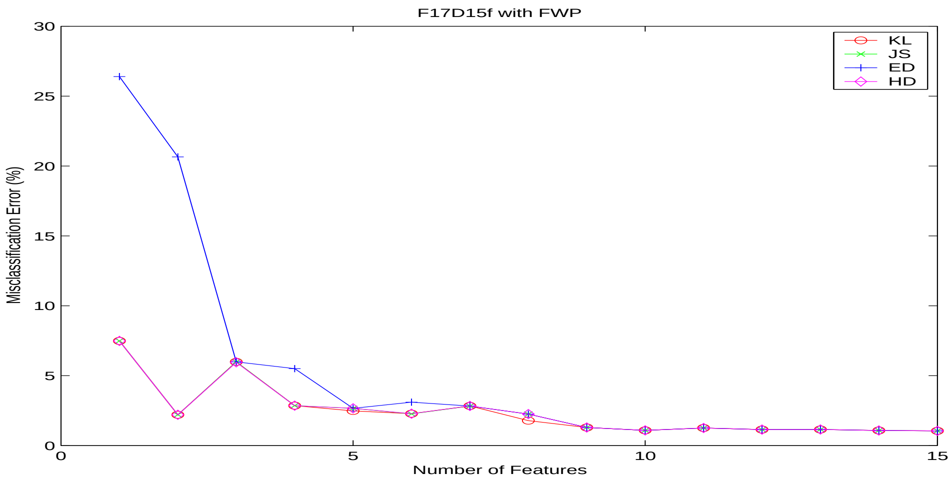

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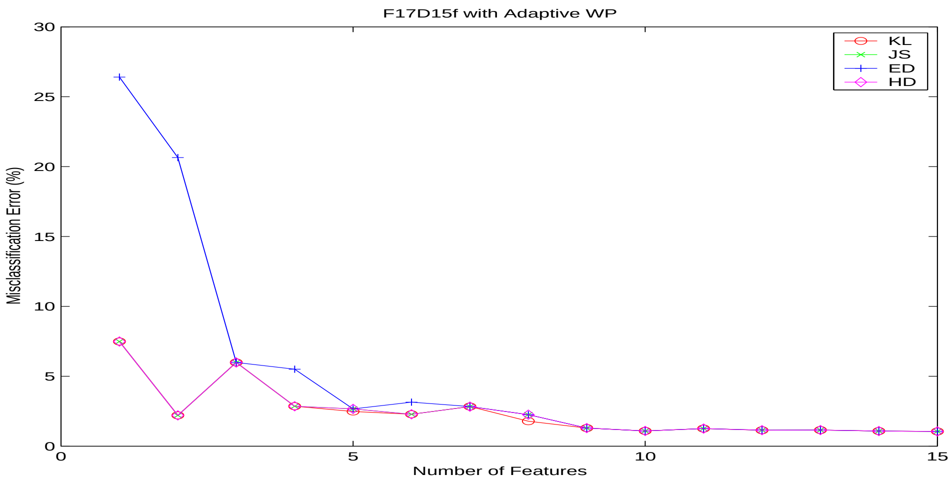

(c)

Figure 3: Classification results for F17D15f
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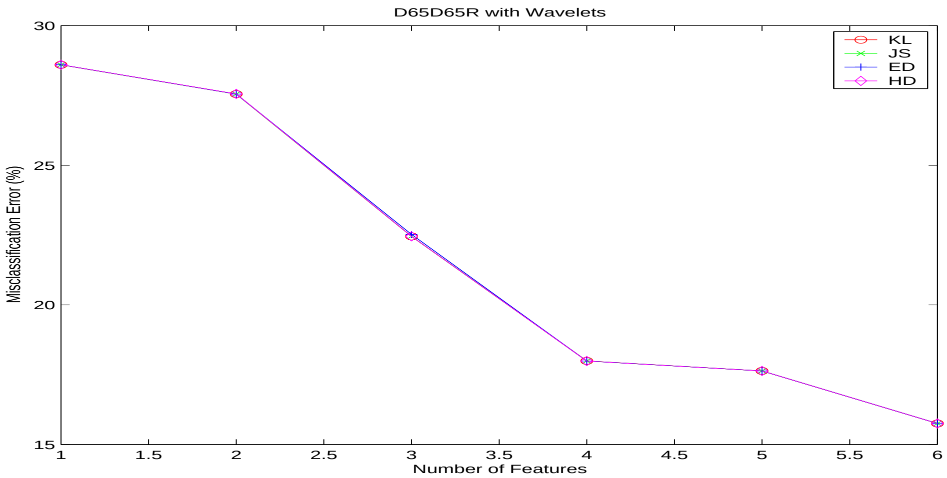

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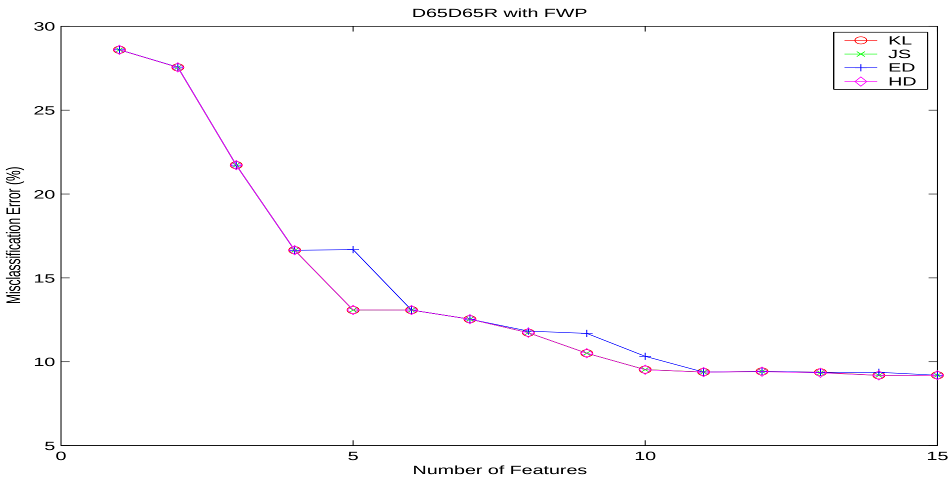

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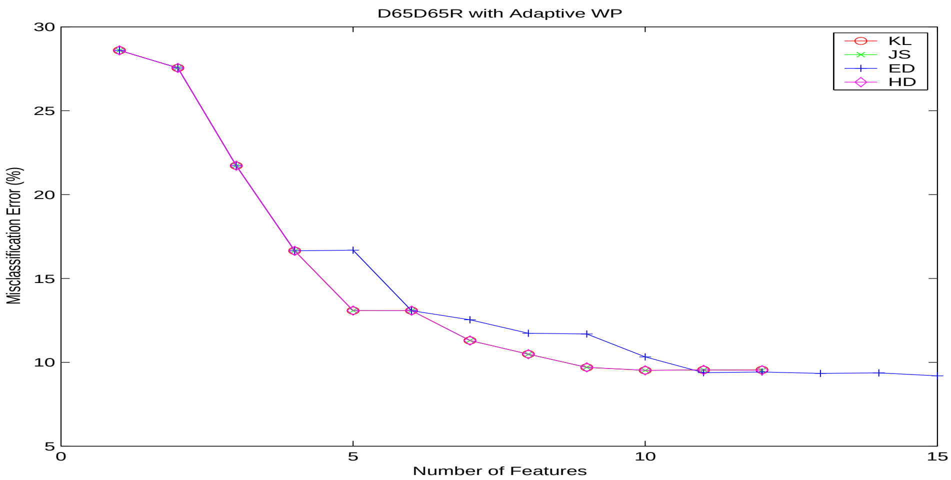

(c)

Figure 4: Classification results for D65D65R
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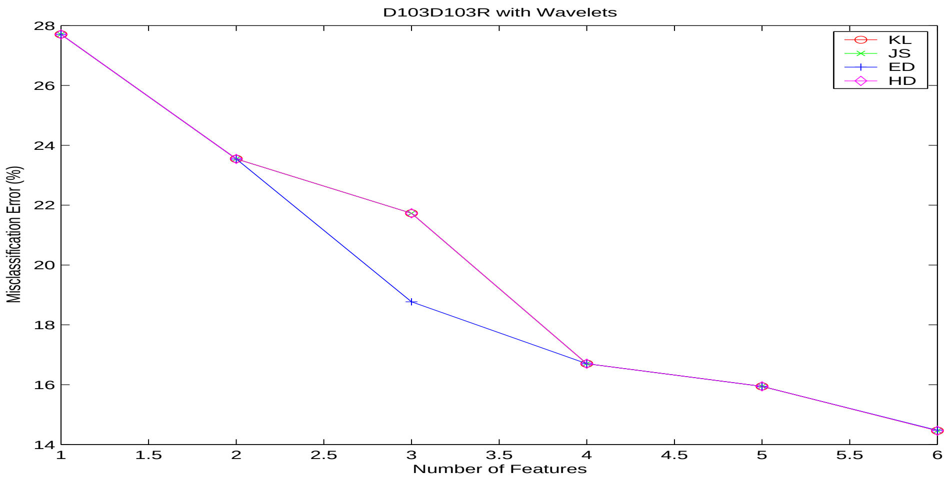

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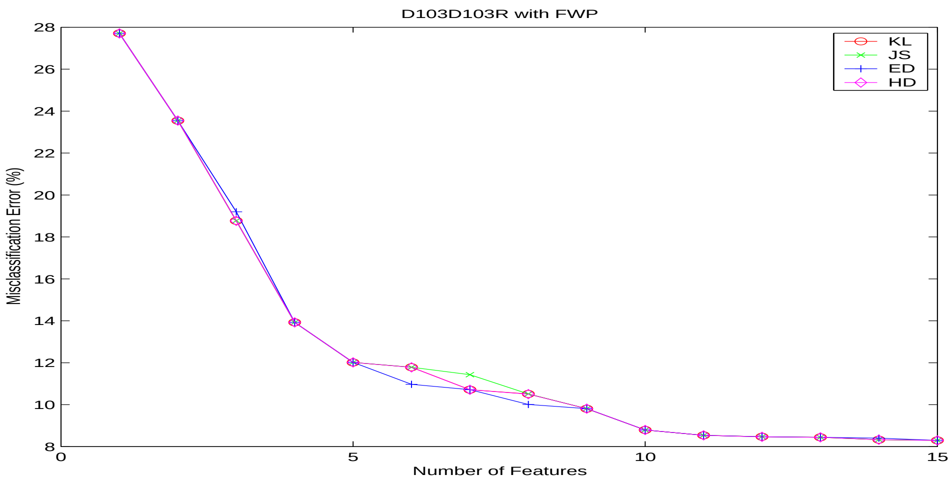

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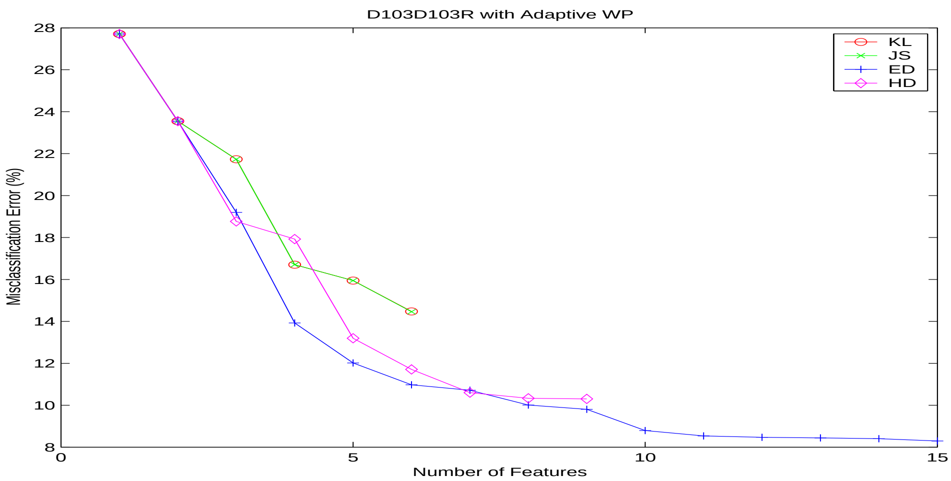

(c)

Figure 5: Classification results for D103D103R
Using (a) Wavelets, (b) FWP, and (c) Adaptive WP